

On some issues of gravitationally induced adiabatic particle productions

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In this work, we investigate the current accelerating universe driven by the gravitationally induced adiabatic matter creation process. To elaborate the underlying cognitive content, here we consider three models of adiabatic particle creation and constrain the model parameters by fitting the models with the Union 2.1 data set using χ^2 minimization technique. The models are analyzed by two geometrical and model independent tests, viz., cosmography and Om -diagnostic, which are widely used to distinguish the cosmological models from Λ CDM. We also compared present values of those model independent parameters with that of the flat Λ CDM model. Finally, the validity of the generalized second law of thermodynamics and the condition of thermodynamic equilibrium for the particle production models have been tested.

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1. INTRODUCTION

Almost two decades have elapsed since the exploitation of the fact that our universe is accelerating by measuring the luminosity distance of the type Ia Supernovae [1]. Since then many observational evidences from different cosmic sources [2–4] have put the opine “universe is accelerating” into a firm observational footing. Current acceleration can be described as an effect of some unknown component(s), named as dark energy for its completely unknown character, and having large negative pressure. Among other dark energy candidates, Λ CDM has been found to depict the contemporary observations at the best. However, this cosmological model suffers from two major drawbacks – cosmological constant problem [5] and the cosmic coincidence problem [6]. These drawbacks forced the grudging cosmologists to tour beyond Λ CDM model. As a consequence, unknown kind of fluid with negative pressure. the dynamical nature of dark energy was proposed. Being the simplest such candidate, the scalar field(s) dark energy models came into existence to explain this late-time cosmic acceleration. As of now there are plenty of scalar field driven models in the literature, such as, quintessence, k-essence, tachyon, phantom, quintom etc (see for instance [7]). These scalar field models are attractive due to its capability to produce cosmic acceleration in agreement with current observational data, but unfortunately, these models can not solve the cosmological constant problem and the cosmic coincidence problem simultaneously. The modified gravity models are also considered for a possible explanation of the current observed universe [8–10] but they suffer from violent instabilities in the cosmic domain [11–13].

Recently, a great attention has been paid on the cosmology of gravitationally induced ‘adiabatic’ particle production since they successfully explain the current accelerated expansion [14–22]. Sometimes, it is claimed that, the entire evolution of the universe is encoded in this mechanism [19, 23, 24]. Also, the authors in Ref. [25] show that particle productions of light non-minimally coupled scalar fields due to the change in the spacetime geometry can lead to an early accelerating universe. Further, it has been argued that the quantum corrections could also lead to particle productions in an early universe which may result in an equation of state $w < -1$ [26–28]. Finally, it has been pointed out that there might have some connection between early inflationary era and the present acceleration of the universe [29]. But here we only concentrate on the late time acceleration of our universe, and we aim to provide a concise description of such a theory in short. We start with recalling the microscopic formulation of particle productions by the gravitational field in 1939 by Schrödinger [30]. After a long period, Parker and collaborators [31] re-investigated this microscopic formulation of particle productions built on the Bogoliubov mode-mixing technique in the background of Quantum Field Theory (QFT) in a curved space-time [32]. After such microscopic investigations of the particle productions, its macroscopic description was studied by Prigogine et al [33] based on the non-equilibrium

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thermodynamics where the universe was assumed to be an open thermodynamical system, and they were successful to connect the particle productions into the Einstein's field equations in a consonant manner. However, this approach was further investigated by a covariant formulation [34] and applied to cosmology where the back reaction term is naturally absorbed into the Einstein's field equations leading to a negative pressure which is responsible for the current cosmic acceleration. However, in connection with this particle productions at the expense of gravitational field of this expanding universe, we recall that long back ago, Zeldovich [35] introduced some bulk viscosity mechanism which is responsible for particle productions. However, later on, Lima and Germano [36] showed that although both the processes, such as, bulk viscosity mechanism by Zeldovich [35] and gravitational particle productions produces same dynamics of the universe but in principle, they are completely different from a thermodynamical point of view. Since we are describing the particle productions, we would also like to note an analogy which exists between the models driven by particle productions and the models of Steady State Cosmology developed in [37]. But, both the approaches are again different from their construction since the Steady State Cosmological models are inspired by adding extra terms into the Einstein-Hilbert action interpreting the so-called C-field, and the creation phenomenon is comprehended through a process of exchanging the energy and momentum between matter itself and the C-field.

In the gravitationally induced particle creation mechanism, the usual balance equation $N_{;\mu}^{\mu} = 0$, is modified as [33]

$$N_{;\mu}^{\mu} \equiv n_{,\mu} u^{\mu} + \Theta n = n\Gamma \iff N_{,\mu} u^{\mu} = \Gamma N, \quad (1)$$

where $N^{\mu} = nu^{\mu}$ is the particle flow vector; u^{μ} is the usual four velocity of the created particles; Γ is the rate of change of the particle number in a physical volume V containing N number of particles, $n = N/V$, is the particle number density, $\Theta = u_{;\mu}^{\mu}$, denotes the fluid expansion. Hence, due to the modifications in the balance equation (1), the field equations $G_{\mu\nu} = 8\pi GT_{\mu\nu}$ will be modified accordingly, and the modified field equations will describe the state of the universe in presence of the creation of particles by the gravitational field. Now, the key of the dynamics is to find an exact functional form of the particle creation rate Γ which could mimic the current observation. But, the possibility of having such a correct functional form of Γ can never be realized until a proper description of QFT in the Friedmann-Lemaître- Robertson-Walker (FLRW) universe is available. Therefore, in general one starts with some choices for the production rate and fit the associated cosmological parameters with the current astronomical data (see for instance [14–22]). Here we have considered three most general phenomenological rate Γ and investigate the evolution equation by Union 2.1 data to see how well they depict present astronomical data. We found that all the models predict a smooth transition from decelerating phase to the current accelerating phase at around $z \sim 1$. Then we have employed two model independent tests, namely, the cosmography and the Om diagnostic so that, we can measure the deviation of the models from the Λ CDM, as this is the best description for our universe till date.

The paper is organized as follows: In section 2, presenting the field equations, we have introduced our three particle production models and analyzed them by Union 2.1 data. In sections 3, 4, we have discussed the model independent tests, cosmography and Om respectively. Then we have presented the thermodynamic analysis of the models in section 5. Finally, in the last section 6, we have discussed the outcomes of our work. We note that throughout the paper, we have used particle productions and matter creation synonymously.

2. FIELD EQUATIONS IN FLRW UNIVERSE

In agreement with cosmological inflation and the cosmic microwave background radiation, the geometry of our universe is very well described by the FLRW line element, which for zero spatial curvature is given by

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2), \quad (2)$$

where $a(t)$ is the scale factor of the universe. In this background, the non-trivial Einstein's field equations for a perfect fluid endowed with gravitationally induced matter creation model are given by

$$H^2 = \frac{8\pi G}{3} \rho, \quad (3)$$

$$\dot{H} + H^2 = -\frac{4\pi G}{3} (\rho + 3p + 3p_c), \quad (4)$$

where ρ , p are respectively the energy density and the thermodynamic pressure of the perfect fluid, p_c denotes the creation pressure due to the gravitationally induced particle productions, and the over dot represents the cosmic time differentiation. Now, under 'adiabatic' condition, this p_c can be written as [23, 33, 34]

$$p_c = -\frac{\Gamma}{3H}(p + \rho), \quad (5)$$

where $H = \dot{a}/a$ is the usual Hubble rate, Γ is the rate of matter creation from the gravitational field. In principle, $\Gamma > 0$ represents the matter creation, $\Gamma < 0$ is for matter annihilation, and $\Gamma = 0$ is the case when there is no matter creation. But, the validity of the generalized thermodynamics in such a scenario induces $\Gamma > 0$. In general, the exact form of Γ is unknown, but it should be determined in the context of quantum processes in curved space time and by taking into account the back reaction effects. Note that, for an expanding universe, the creation pressure p_c is negative.

In what follows, we consider that the perfect fluid satisfying the barotropic equation of state

$$p = w\rho, \quad (6)$$

where $w \geq 0$ is the equation of state parameter of the perfect fluid. Thus, $w = 1/3$ represents the radiation dominated era, whereas $w = 0$ stands for non-relativistic matter. Now, combining the Einstein's field equations (3), (4) and the barotropic equation of state in Eq. (6), we find

$$\dot{\rho} + 3H(1+w) \left(1 - \frac{\Gamma}{3H}\right) \rho = 0, \quad (7)$$

which is nothing but the energy conservation equation and could also be obtained directly from the Bianchi's identity $T^{\mu\nu}_{;\nu} = 0$ (where $T^{\mu\nu} = (\rho + p + p_c)u^\mu u^\nu + (p + p_c)g^{\mu\nu}$). However, for $\Gamma \ll 3H$, we recover the original energy conservation equation $\dot{\rho} + 3H(1+w)\rho = 0$, showing no matter creation effect. Now, combining (3), (6) and (7), we have the following evolution equation

$$\frac{dH}{dt} + \frac{3}{2}(1+w)H^2 \left(1 - \frac{\Gamma}{\Theta}\right) = 0, \quad (8)$$

and the deceleration parameter which measures the decelerating/accelerating phase of the universe takes the form

$$q = - \left(1 + \frac{\dot{H}}{H^2}\right) = -1 + \frac{3}{2}(1+w) \left(1 - \frac{\Gamma}{\Theta}\right). \quad (9)$$

Now, the dynamics of the universe can only be surveyed after the particle creation rate Γ is known. The particle production rate is related to the nature of the produced particles under this adiabatic mechanism, and unfortunately, the nature of the produced particles is unknown to us as the associated QFT is yet to be developed which is an essential tool to determine this Γ . But, as particle production mechanism has become one of the possible alternative to explain the current expanding accelerating phase of the universe, so we start with some phenomenological but general choices for Γ . It has been shown that, $\Gamma \propto H^2$ [38, 39] gives the inflationary solution, $\Gamma \propto H$ [40] can explain the intermediate decelerating era, and even $\Gamma = \text{constant}$, can explain the evolution of the universe from big bang to the present accelerating stage [23]. Further, a linear combination of the above rates [41] predicts two accelerating phases of the universe, one at early phase of its evolution and the second one stands for the present day acceleration. Furthermore, the same linear combination of the above rates claims the decelerating nature of our universe in future [19], as also predicted in some other contexts [42, 43]. Therefore, we begin with some phenomenological but general choices for Γ of the form $\Gamma = 3\beta H f(z)$ where $f(z)$ is any arbitrary function of the redshift z , and β is any free non-negative parameter describing the rate of the creation. In order to investigate their viabilities on the onset of late-time accelerating phase of the universe. Hence, we start with the following three choices for Γ :

$$\text{Model I:} \quad \Gamma = \frac{3\beta H}{1+z} \tanh\left(\frac{\alpha}{1+z}\right), \quad (10)$$

$$\text{Model II:} \quad \Gamma = \frac{3\beta H}{1+z} \tanh(\alpha(1+z)), \quad (11)$$

$$\text{Model III:} \quad \Gamma = 3\beta H \left(\frac{1}{1+z}\right). \quad (12)$$

These models contain only two parameters α , and β . We note that, at a particular redshift, the creation rate is solely dependent on β , although we have another parameter, α , but this does not play any significant role to increase or decrease the particle creation rate due to very slowly varying nature of the function $\tanh(x)$ ($x \in \mathbb{R}$). Further, we note that, it is not possible for β to be infinitely large, because in that case, the production of particles will be very large and the evolution of cold dark matter particles could exceed the standard evolution law a^{-3} , which will contradict the present observations. Hence, the only restriction on the particle creation models is that, the free parameter β should take its value in such a way so that $\Gamma/3H \leq 1$. In the following subsections we discuss the pros and cons of the above models.

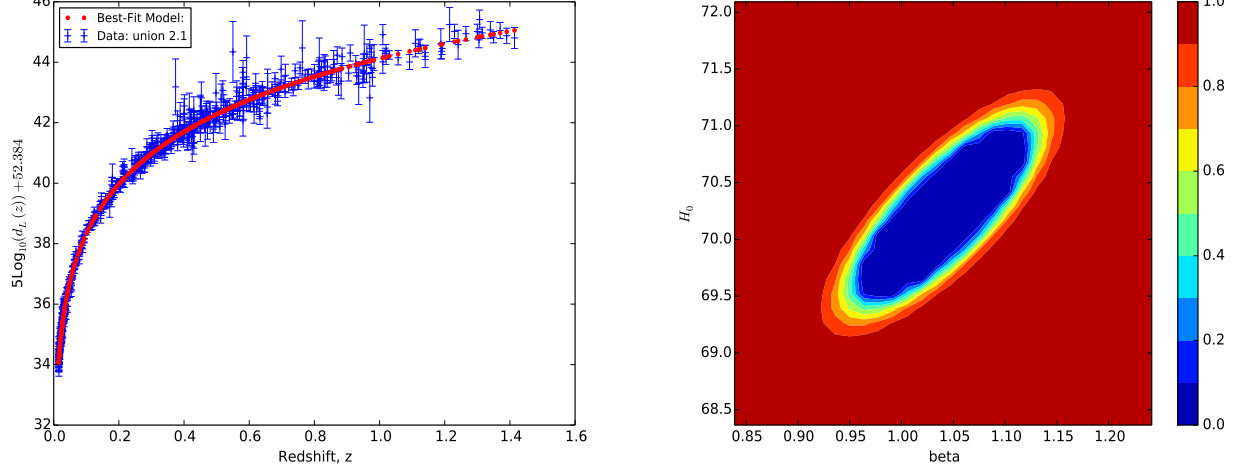


FIG. 1: Model I for best fit H_0 and β along with the Union 2.1 data

2.1. Model I: $\Gamma = \frac{3\beta H}{1+z} \tanh\left(\frac{\alpha}{1+z}\right)$, where $\beta > 0$, $\alpha > 0$ are any real numbers

Solving the evolution equation (8) for this choice of Γ , we find

$$H = H_0(1+z)^{\frac{3}{2}(1+w)} \exp\left(-\frac{3}{2}(1+w)\beta \int_0^z \frac{\tanh\left(\frac{\alpha}{1+z}\right)}{(1+z)^2} dz\right). \quad (13)$$

the analytic solution of which can be found as

$$H(z) = H_0(1+z)^{\frac{3}{2}(1+w)} \left(\frac{\exp\left(\frac{2\alpha}{1+z}\right) + 1}{\exp(2\alpha) + 1} \right) \exp\left(\frac{3}{2}(1+w) \frac{\beta z}{1+z}\right). \quad (14)$$

For our analysis we have used χ^2 minimization technique keeping α fixed at 1 and vary H_0 and β . With the help of Union 2.1 data alone, we find that $H_0 = 69.9352 \text{ Km S}^{-1} \text{ Mpc}^{-1}$, and $\beta = 1.0403$ provide the best fit for Model I. The best fit H_0 and β corresponds to reduced Chi-square of $\chi^2_{\text{reduced}} = 0.9698$.

2.2. Model II: $\Gamma = \frac{3\beta H}{(1+z)} \tanh(\alpha(1+z))$, where $\beta > 0$, $\alpha > 0$ are any real numbers

Again solving the Eq. (8), the Hubble parameter can be written as

$$H = H_0(1+z)^{\frac{3}{2}(1+w)} \exp\left(-\frac{3}{2}(1+w)\beta \int_0^z \frac{\tanh(\alpha(1+z))}{(1+z)^2} dz\right). \quad (15)$$

Here, also we have kept α fixed at 1 and vary only H_0 and β . The model is best fitted with Union 2.1 data set for $H_0 = 69.8925 \text{ Km S}^{-1} \text{ Mpc}^{-1}$ and $\beta = 1.0235$ with reduced Chi-square of $\chi^2_{\text{reduced}} = 0.9696$.

2.3. Model III: $\Gamma = 3\beta H\left(\frac{1}{1+z}\right)$

In this model, we have only one parameter β . As before solving the Eq.(8) we obtain

$$H = H_0(1+z)^{\frac{3}{2}(1+w)} \exp\left(-\frac{3\beta}{2}(1+w) \frac{z}{1+z}\right) \quad (16)$$

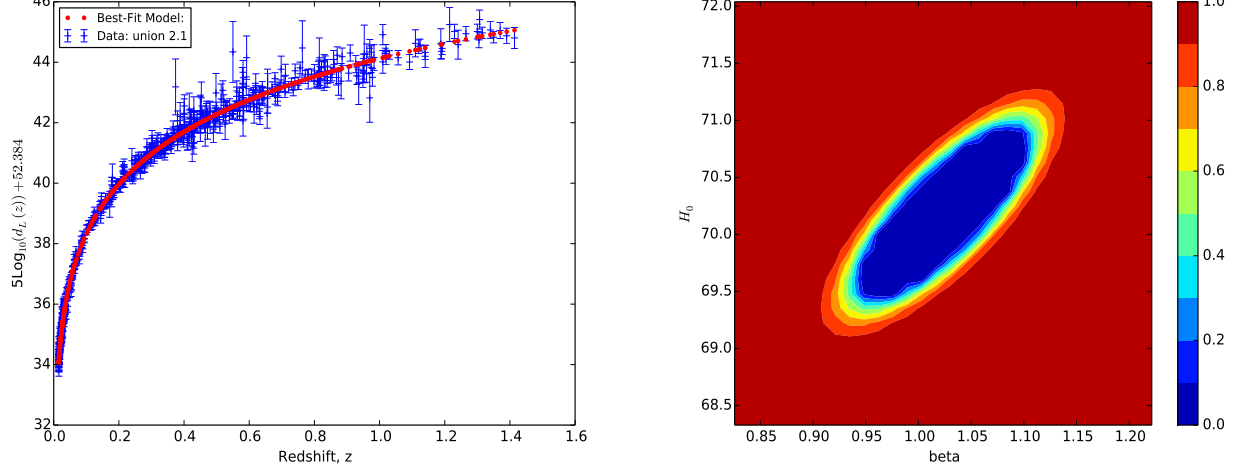


FIG. 2: Model II for best fit H_0 and β along with the Union 2.1 data.

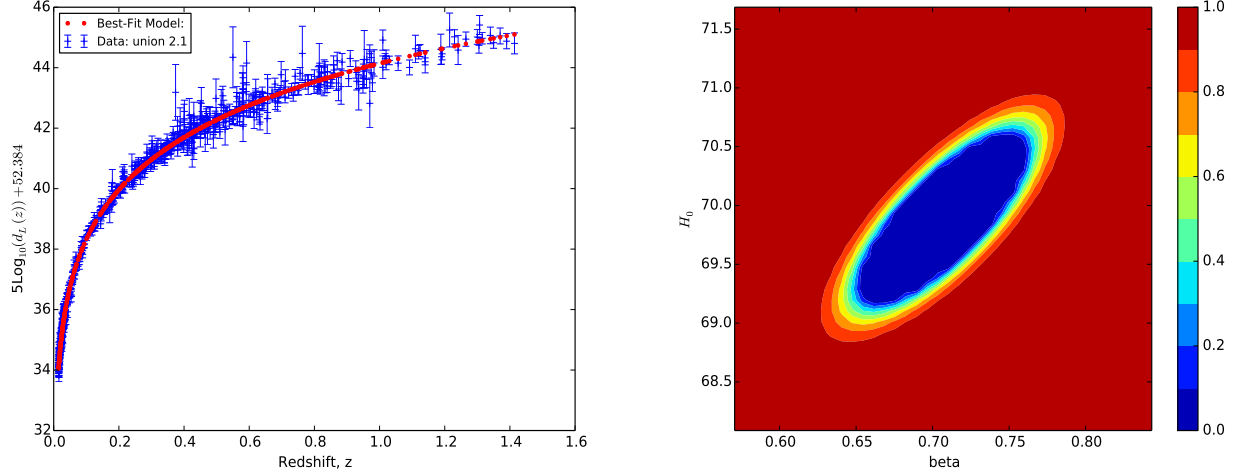


FIG. 3: Model III for best fit H_0 and β along with the Union 2.1 data.

The model is best fitted with Union 2.1 data set for $H_0 = 69.88627 \text{ Km S}^{-1} \text{ Mpc}^{-1}$ and $\beta = 0.70693$ with reduced Chi-square $\chi^2_{\text{reduced}} = 0.97$.

Figures 1, 2, 3 display the errors bars (left panel) with the best fit values in the plane (H_0, β) (right panel) with 1σ , 2σ confidence levels. In figure 4, we have shown the evolution of the deceleration parameters for three particle creation models along with the flat Λ CDM model, where the models I and II predict the transition of the decelerating phase to the present accelerating phase at around $z \sim 0.6$ (transition redshift of Λ CDM), whereas model III predicts the transition at around $z \sim 1.1$.

3. MODEL INDEPENDENT TEST I: COSMOGRAPHY

Model independent description is always fascinating for any kind of studies in nature, and when the discussion deals with our current accelerating universe, specifically with the dark energy models, it becomes essential due to the large number of dark energy models in the literature. Two notable model independent geometrical, dimensionless

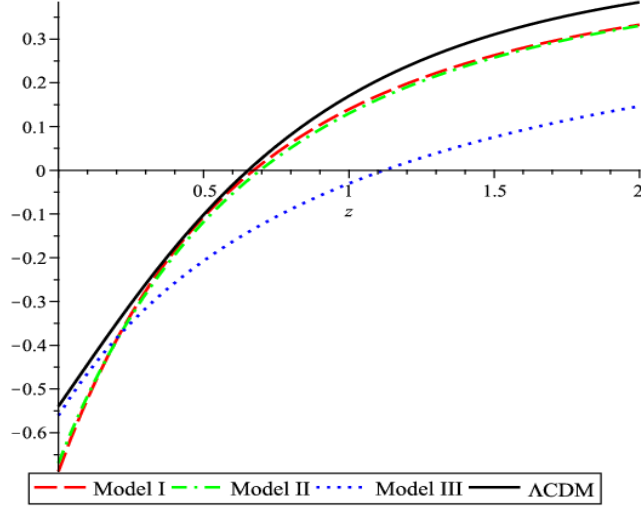


FIG. 4: The plot displays the transition of the deceleration parameters for three models in compared to the flat Λ CDM model.

parameters $\{r, s\}$ are defined as [44]

$$r = \frac{1}{aH^3} \frac{d^3 a}{dt^3}, \quad \text{and} \quad s = \frac{r-1}{3\left(q - \frac{1}{2}\right)}, \quad (17)$$

where a , H , q have their usual meanings. The parameters are used to compare the goodness of several dark energy models with the Λ CDM model, where for the flat Λ CDM, $\{r, s\} = \{1, 0\}$. That means, throughout the evolution of the universe, $r(z_1) - r(z_2) = 0$, for any two arbitrary redshifts z_1, z_2 . Later this model independent approach was further extended in Ref. [45] by considering the Taylor series expansion of the scale factor around the present time, and, which give rise some new model independent dimensionless geometrical parameters as follows:

$$j = \frac{1}{aH^3} \frac{d^3 a}{dt^3}, \quad s = \frac{1}{aH^4} \frac{d^4 a}{dt^4}, \quad l = \frac{1}{aH^5} \frac{d^5 a}{dt^5}, \quad \text{and} \quad m = \frac{1}{aH^6} \frac{d^6 a}{dt^6}. \quad (18)$$

The parameters in (18) together with H and q are called the cosmographic parameters¹. The cosmographic parameters in Eq. (18) can be rewritten as

$$j = -q + (1+z) \frac{dq}{dz} + 2q(1+q), \quad (19)$$

$$s = j - 3j(1+q) - (1+z) \frac{dj}{dz}, \quad (20)$$

$$l = s - 4s(1+q) - (1+z) \frac{ds}{dz}, \quad (21)$$

$$m = l - 5l(1+q) - (1+z) \frac{dl}{dz}. \quad (22)$$

Therefore, from the above set of equations, we may argue that, as long as the Hubble parameter for any cosmological model is fourth order differentiable, the cosmography of that model will be valid.

In Figure 5, we have shown the evolution of the cosmographic parameters for three particle creation models. In all the plots, we have kept the evolution of the corresponding cosmographic parameters for Λ CDM in order to compare the phenomenological models. From the figures (even the table I also reflects the same nature), we find that, the parameters j, l have similar evolution. On the other hand, the remaining parameters s, m predict an equivalent evolution.

¹ Here r and j are same; but the s parameter defined in (18) is not same with one defined in (17)

Model	q_0	j_0	s_0	l_0	m_0
Λ CDM	-0.53665	1.00000	-0.39005	3.21486	-11.49597
Model I	-0.56035	1.12798	2.01690	8.17418	23.78283
Model II	-0.54115	0.93308	0.98352	3.37609	1.68084
Model III	-0.56039	1.12809	2.01731	8.17558	23.79135

TABLE I: The table shows the present values of deceleration (q_0) and all cosmographic parameters for the three models along with the flat Λ CDM model.

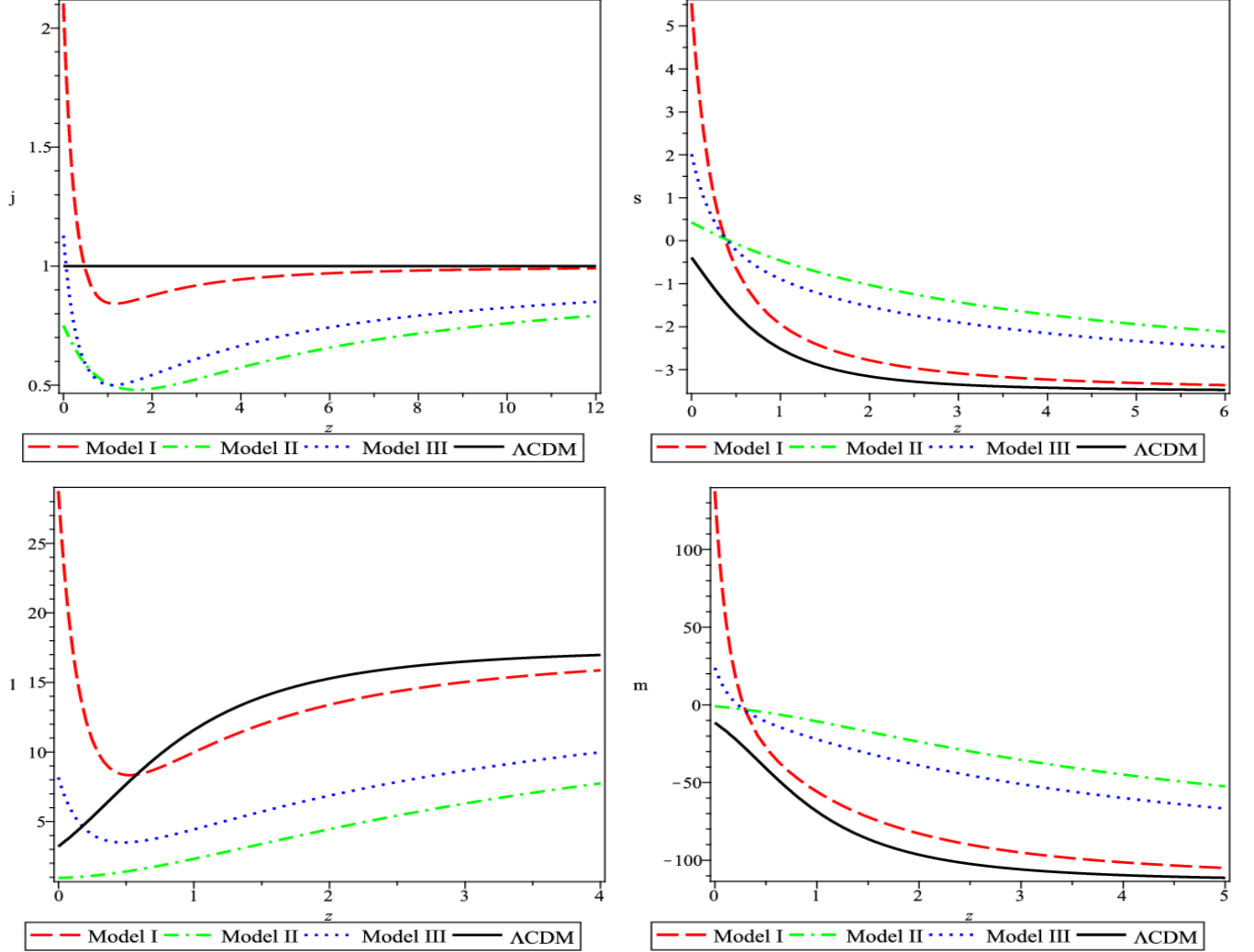


FIG. 5: The plots depict the evolution of the cosmographic parameters j , s , l , m for three models in compared to the flat Λ CDM model

4. MODEL INDEPENDENT TEST II: THE DIAGNOSTIC Om AND ITS IMPROVED VERSION

Through the statefinder parameters and its extension cosmography, we are able to distinguish between several theoretically developed dark energy models from the Λ CDM model. In 2008, another model independent test Om was introduced [46] which can also differentiate the dark energy models from Λ CDM. The Om function is defined as [46]

$$Om(z) = \frac{\tilde{h}^2(z) - 1}{(1+z)^3 - 1}, \quad \text{where } \tilde{h} = \frac{H(z)}{H_0}. \quad (23)$$

$Om h^2(z_i; z_j)$	Λ CDM	Model I	Model II	Model III
$Om h^2(z_1; z_2)$	0.14170 ± 0.00097	0.05729	0.05696	0.05762
$Om h^2(z_1; z_3)$	0.14170 ± 0.00097	0.02892	0.02831	0.02906
$Om h^2(z_2; z_3)$	0.14170 ± 0.00097	0.01576	0.01521	0.01582

TABLE II: The table shows the $Om h^2$ values for the models I, II, and III in compared to the flat Λ CDM model.

Eq. (23) is very simple and elegant. It needs only the expansion rate to find the distinction of any dark energy model from the Λ CDM. It is easy to see that, for a spatially flat Λ CDM model, Eq. (23) reduces to $Om(z) = \Omega_{m0}$ (where Ω_{m0} is the density parameter for the cold dark matter). That means, this function stays constant for flat Λ CDM model throughout the entire evolution of the universe. So, essentially, for a flat Λ CDM model, $Om(z_1) - Om(z_2) = 0$, for any two redshifts z_1, z_2 . Therefore, compared to the statefinder parameter ‘ r ’, the behavior of Om is almost same as both of them stay constant for flat Λ CDM, but, there is one notable property we should mention. r needs triple derivative term with respect to the cosmic time, whereas Om is related to the expansion history, that means, one time derivative with respect to the cosmic time. So, essentially, Om is a more easier geometrical test in compared to the statefinders and cosmography. However, we can see Om as a two point function [47] in the following way

$$Om(z_i; z_j) = \frac{\tilde{h}^2(z_i) - \tilde{h}^2(z_j)}{(1+z_i)^3 - (1+z_j)^3}. \quad (24)$$

Basically, Eq. (24) is nothing but (23). If we simply put $z_j = 0$ in (24), we see $Om(z_i; 0) = Om(z_i)$ which is Eq. (23). It is worthy to mention the elegant nature of Eq. (24). If we can know the values of the Hubble parameter at two or more redshifts, we can reconstruct Om directly from the observations, and, consequently, this reconstruction will surely tells us whether the present universe is dominated by Λ CDM or not. However, Eq. (24) can be written in a sophisticated way by multiplying both sides of it by h^2 (where $h = H_0/100\text{km/sec/Mpc}$) as follows [48]

$$Om h^2(z_i; z_j) = \frac{h^2(z_i) - h^2(z_j)}{(1+z_i)^3 - (1+z_j)^3}. \quad (25)$$

where $h(z) = H(z)/100\text{km/sec/Mpc}$, and, $i \neq j$. For flat Λ CDM model, we get

$$Om h^2 = \Omega_{m0} h^2. \quad (26)$$

The latest Planck mission [49] claims that, $\Omega_{m0} h^2 = 0.14170 \pm 0.00097$ (TT,TE,EE+lowP+lensing+ext). Thus, for the flat Λ CDM model, we have

$$Om h^2 = 0.14170 \pm 0.00097, \quad (27)$$

which is nothing but an indication to those dark energy candidates which try to deviate from flat Λ CDM. Let us now consider three redshifts $z_1 = 0$, $z_2 = 0.57$, $z_3 = 2.34$, out of which the measurement of the Hubble parameters at 0.57, 2.34 are statistically independent. Thus, they will be very much helpful in order to calculate $Om h^2(z_i; z_j)$.

5. THERMODYNAMIC RESTRICTIONS

We devote this section in order to check the conditions of thermodynamic equilibrium for the present particle production models. Since from physical point of view, the macroscopic systems tend toward the thermodynamic equilibrium and the entropy (S) of an macroscopic system is never decreasing. So, from mathematical point of view, if S is the total entropy of the macroscopic system, then we must have $\dot{S} \geq 0$ (Entropy never decreasing), and $\ddot{S} < 0$ (Equilibrium condition) for $t \rightarrow \infty$. In other words, the entropy should be concave in a small neighborhood of the equilibrium point. Now, the total entropy (S) is contributed from the entropy of the apparent horizon (S_h) and the entropy of the fluid (S_w) with the equation of state $p = w\rho$, that means, essentially, we have to consider the behavior of $S = S_h + S_w$. Now, the entropy of the apparent horizon is given by $S_h = k_B \mathcal{A}/4l_{pl}^2$, where k_B is the Boltzmann’s constant, $\mathcal{A} = 4\pi r_h^2$, is the horizon area in which $r_h = (H^2 + k/a^2)^{-1/2}$ is the horizon radius [50] which

finally becomes $r_h = 1/H$. Now, differentiating S_h with respect to the cosmic time and remembering the fact that we are considering flat FLRW universe, one gets

$$\dot{S}_h = - \left(\frac{2\pi k_B}{l_{pl}^2 H^3} \right) \dot{H} = \frac{3\pi k_B}{l_{pl}^2 H} (1+w) \left(1 - \frac{\Gamma}{3H} \right), \quad (28)$$

which shows that for $\dot{S}_h \geq 0$, we should have $\Gamma/3H \leq 1$. Now, we recall the Gibb's equation for the fluid which is a relation between the thermodynamic quantities associated with the fluid as $TdS_w = d(\rho V) + pdV$, where by S_w we mean the entropy of the fluid, $V = 4\pi r_h^3/3$ is the volume of the region surrounded by the radius r_h , and T is the fluid temperature. Now, encountering the cosmic time in the Gibb's equation one may come at

$$T\dot{S}_w = 6(1+w)\pi \left(1 - \frac{\Gamma}{3H} \right) (1+3w). \quad (29)$$

Since for $w > 0$, we must have that $\dot{S}_w \geq 0$. Finally, one has that $\dot{S}_h + \dot{S}_w \geq 0$ for $\Gamma/3H \leq 1$. So, the entropy of the horizon plus the fluid is an increasing function of the cosmic time. Now, we proceed for the equilibrium condition, and we need to define the temperature of the fluid first. If one assumes that the temperature of the fluid becomes equal to that of the temperature of the horizon defined as $T_h = 1/2\pi r_h$ [51] then one may find the second derivatives of S_w . However, differentiating \dot{S}_h again with respect to the cosmic time, we find

$$\ddot{S}_h = \left(\frac{3\pi k_B(1+w)}{l_{pl}^2} \right) \left[\frac{3}{2}(1+w) \left(1 - \frac{\Gamma}{3H} \right) \left(1 - \frac{2\Gamma}{3H} \right) - \frac{1}{3H} \frac{\dot{\Gamma}}{H^2} \right]. \quad (30)$$

Similarly, if one differentiates eq. (29) with respect to the cosmic time, then one gets

$$\ddot{S}_w = 12\pi^2(1+w)(1+3w) \left[\frac{3}{2}(1+w) \left(1 - \frac{\Gamma}{3H} \right) \left(1 - \frac{2\Gamma}{3H} \right) - \frac{1}{3H} \frac{\dot{\Gamma}}{H^2} \right]. \quad (31)$$

Now, adding equations (30) and (31), one has

$$\ddot{S}_h + \ddot{S}_w = \left(\frac{3\pi k_B(1+w)}{l_{pl}^2} + 12\pi^2(1+w)(1+3w) \right) \left[\frac{3}{2}(1+w) \left(1 - \frac{\Gamma}{3H} \right) \left(1 - \frac{2\Gamma}{3H} \right) - \frac{1}{3H} \frac{\dot{\Gamma}}{H^2} \right]. \quad (32)$$

Now, the key role in determining the sign of $S = \ddot{S}_h + \ddot{S}_w$ is played by the second bracketed term in equation (32). If one proceeds further, then after some simple steps, one may conclude that $\ddot{S} < 0$, provided the rate Γ satisfies the following simple relation

$$\frac{d\Gamma}{dH} + 3H \left(1 - \frac{2\Gamma}{3H} \right) < 0, \quad (33)$$

which can be considered as a very general relation for all particle production models. From here, since we have introduced the rate of change of production rate $\Gamma' \equiv \frac{d\Gamma}{dH}$, so one may encounter with two possible cases as follows: It may happen that either $\Gamma' > 0$, or $\Gamma' < 0$. However, one may notice that Γ/H plays an important role since we have already noticed that this term deviates the evolution equations from their standard laws, and also for $\Gamma/3H \ll 1$, one gets back the standard evolution, so we express the inequality in eq. (33) by the following inequality

$$\frac{d}{dt} \left(\frac{\Gamma}{H} \right) > \frac{3}{2} (1+w) \left(1 - \frac{\Gamma}{3H} \right) \left[\frac{\Gamma}{H} + 3H \left(1 - \frac{2\Gamma}{3H} \right) \right]. \quad (34)$$

The prescription is very general in the sense that it provides a thermodynamic constraints over any particle production model. Since we have considered three different particle production models given in (10), (11), and (12), we find that the generalized second law of thermodynamics is always valid for $\Gamma/3H < 1$, and the models will be

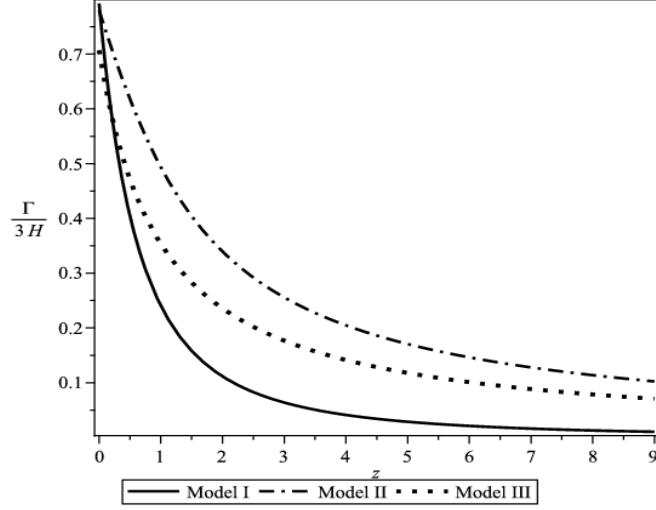


FIG. 6: For the best fit value of β , for all models, we show that the ratio $\Gamma/3H$ does not exceed 1 at present time (i.e. $z = 0$) which proves the validity of the generalized second law of thermodynamics.

thermodynamically stable if one of the inequalities in equations (33), (34) holds good.

Now, we shall investigate the same thermodynamical properties when the temperature of the fluid is governed by the following law [52, 53]

$$\frac{\dot{T}}{T} = 3H \left(\frac{\Gamma}{3H} - 1 \right) \frac{\partial p}{\partial \rho}. \quad (35)$$

Since for our model $\frac{\partial p}{\partial \rho} = w$, so using the evolution equation (8) one can solve the above equation (35) as

$$T = T_0 \left(\frac{H}{H_0} \right)^{2w/(1+w)}, \quad (36)$$

where T_0 , H_0 are respectively the present values of the temperature and the Hubble parameter. One may note that the new temperature in (36) could effect on the equilibrium condition not on the generalized second law of thermodynamics, since the temperature is positive as observed from equation (36), so from equations (28), (29), it is clear that both \dot{S}_h and \dot{S}_w are non-negative for $\Gamma/3H \leq 1$, thus their addition too, hence the generalized second law of thermodynamics is valid. Now, we need to calculate only \ddot{S}_w since \ddot{S}_h will remain same as it is in equation (30). So, using the temperature (36) into equation (29), one may find that

$$\ddot{S}_w = -\frac{6\pi}{T_0} (1+w)(1+3w) \left[\frac{3H}{2}(1+3w) \left(1 - \frac{\Gamma}{3H} \right) \left(\frac{\Gamma}{3H} - \frac{2w}{1+w} \right) + \frac{\dot{\Gamma}}{3H} \right]. \quad (37)$$

Although it seems difficult to arrive at some conclusion from the sum $\ddot{S}_h + \ddot{S}_w$ on the thermodynamic equilibrium, but we find that under the following conditions the model could reach the thermodynamic equilibrium.

1. If the inequality (33) holds good together with (i) $\dot{\Gamma} < 0$, and (ii) $\Gamma/3H < 2w/(1+w)$.
2. Finally, the inequality (33) should also again hold and

$$\frac{d\Gamma}{dH} < \left(\frac{1+3w}{1+w} \right) \left(\frac{\Gamma}{3H} - \frac{2w}{1+w} \right). \quad (38)$$

Thus, we find that under some suitable conditions described above, the generalized second law of thermodynamics holds good for $\Gamma/3H \leq 1$ irrespective of the temperature of the fluid, but on the other hand, the thermodynamic equilibrium is reached under the specified conditions which significantly depends on the temperature as we have considered in this section. So, the generalized second law of thermodynamics holds for models I, II, III provided $\Gamma/3H \leq 1$.

In figure 6, we have graphically presented the behavior of the $\Gamma/3H$ for all models for the best fit value of β constrained from union 2.1. We find that this ratio does not exceed ‘1’, which means the the generalized second law of thermodynamics is hold. As far as the equilibrium is concerned, it becomes difficult to infer without the above prescribed relations.

6. SUMMARY

In this work, we have investigated the present day accelerating universe driven by the gravitationally induced adiabatic particle productions. To illustrate this notion, we have introduced three phenomenological but generalized models of particle production where the particle production rate Γ is a function of the Hubble parameter and the redshift in an explicit manner. We derived the analytic expressions for the Hubble parameters and constrained the model parameters using Union 2.1 data alone. Then, the models were confronted with two model independent tests, namely, cosmography and Om . In cosmographic analysis we found that, at late-time, all the models agree with the Λ CDM up to the jerk parameter, but from snap to m parameter, we observe an unusual behavior, even in signs of the parameter values in compared to those of the flat Λ CDM model. In Om diagnostic, we evaluated $Om h^2$ at three different points. The values of $Om h^2$ as evaluated from the models however are different from that of the flat Λ CDM, which indicates that our models do not fall into the Λ CDM category. So, from the above analysis, it is very clear that the models predict a similar behavior to Λ CDM up to jerk parameter. Even though the models I and II transit from deceleration to present accelerating phase at around $z \sim 0.6$. But, when we bring into the Om diagnostic, we see that the models predict a considerable deviation from Λ CDM. Therefore, we found that the phenomenological models do not work simultaneously with both the model independent tests while they have some other interesting features which mimic current observational data. Finally, we analyzed the validity of the generalized second law of thermodynamics for three models. We found that the models are in agreement with this generalized second law when $\Gamma/3H \leq 1$, and this is supported by our observational analysis. But, we could not reach any definite conclusion for thermodynamic equilibrium except for a very general analysis given in equations (33), (34) and (38).

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